

# Next-to-leading Order Parton Model Calculations in the Massless Schwinger Model

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## ABSTRACT

We carry out next-to-leading order (NLO) parton model calculations for the standard hard “QCD” processes in the massless Schwinger model. The asymptotic expansion of the exact result for the deep inelastic cross section is used to infer the NLO distribution function. These distribution functions are then used to calculate the NLO Drell-Yan parton model cross section and it is found to agree with the corresponding term in the expansion of the exact result for the Drell-Yan cross section. Finally, by using the bosonization formula and the exact solutions we study the interference between different partonic processes.

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# 1 Introduction

Although the parton model [1, 2] lacks a solid theoretical foundation, it is an essential tool in QCD applications to strong interactions and so far it has been reasonably successful. If we have a theory that exhibits many properties of 4-dimensional quantum chromodynamics, QCD<sub>4</sub>, and this theory is exactly solvable, then it is important that we study how well parton model works in this theory. The massless Schwinger model is one such theory in that it is an exactly solvable, interacting quantum field theory [3, 4] that is both asymptotically free and exhibits confinement [5, 6].

In reference [7], the exact cross sections for lepton-antilepton annihilation, deep inelastic scattering and Drell-Yan processes in the massless Schwinger model coupled to a scalar current are calculated in terms of the functions  $R_{\pm}(q^2)$ . In [8] the full asymptotic expansions of  $R_{\pm}(q^2)$  were described and the terms up to and including order  $(g^2/q^2)^4$  were explicitly computed where  $g$  is the strong coupling constant and  $q^2$  is the squared momentum transfer. The leading terms in these exact cross sections were then shown to equal the leading order parton model results in the Bjorken scaling region. This paper extends this analysis of the massless Schwinger model to the next-to-leading order (NLO).

Of course, the lack of transverse momenta for the Schwinger model partons and the corresponding absence of jets limits the extent of the lessons that we adduce for QCD<sub>4</sub>. Another fundamental difference between the Schwinger model and QCD<sub>4</sub> is that the Schwinger model coupling constant  $g$  has the dimensions of mass. However, we see this difference as an opportunity to consider several potential problems for the parton model when it is pushed to next order in  $g^2/q^2$  including: (1) the mass of the hadron which is of order  $g$  in this model can no longer be neglected in the kinematics; (2) hadronization or bound state effects are assumed to be suppressed by inverse powers of  $q^2$  and so may arise at order  $g^2/q^2$ ; (3) quantum interference of the hard partonic processes are normally assumed to be power suppressed and so may arise at order  $g^2/q^2$ ; and finally (4) the analogue of higher twist terms.

To our surprise, despite these complications, we find that the parton model NLO Drell-Yan cross section agrees with the exact Drell-Yan cross section at NLO. We also argue that at NLO the Schwinger model photon, the analogue in our model of the gluon, has a parton distribution, which may be calculated by evaluating processes at order  $g^4/q^4$ . As another application, we use the bosonization formula to isolate the contributions coming from the interference of the underlying partonic processes. Then from the expansion of the exact results, we find that the interference is suppressed by order  $(g^2/q^2)^4$ .

## 2 NLO Parton Distribution Functions From Deep Inelastic Scattering

The processes that we calculate occur in the same model used in [7, 8, 5, 6]. For completeness we briefly describe the model here. We extend the massless Schwinger model [3] by including

a massless fermion  $f$  (our “lepton”) that is not QED<sub>2</sub>-charged but interacts with the QED<sub>2</sub>-charged fermions,  $\psi$ , through a Yukawa coupling to a scalar photon  $\phi$ . The full Lagrangian is [5, 6]

$$L = F^2/4 + \bar{\psi}(i \not{\partial} + g \not{A})\psi + \bar{f}i \not{\partial}f + 1/2\phi\Box\phi + e(\bar{\psi}\psi + \bar{f}f)\phi. \quad (1)$$

So  $e$  is the analogue of the electromagnetic coupling, and  $g$  is the analogue of the QCD<sub>4</sub> strong coupling constant so from now on, we will call the photon of QED<sub>2</sub> as “gluon”. All of our calculations are done at lowest order in  $e$ . Then, using the dual realization of the Schwinger model in terms of a free scalar of mass (which we will call “hadron”)  $m_h = g/\sqrt{\pi}$  and the bosonization formula <sup>4</sup>

$$\bar{\psi}\psi = c : \cos(2\sqrt{\pi}h) :, \quad (2)$$

we may compute the process cross sections exactly in  $g$ .

First consider the DIS process

$$f(k_1) + h(P) \rightarrow f(k_2) + X,$$

where  $h$  is the target particle and the final states  $X$  are summed over and where the momentum assignments are given in the parantheses. The cross section at leading order in  $e^2$ , but all orders in  $g^2$ , is

$$d\sigma = \frac{e^4}{q^4} \frac{1}{2E_P} R_5((q+P)^2) dk'. \quad (3)$$

where  $q = k_1 - k_2$  is the momentum transfer, and  $P$  is the hadron momentum,  $R_5(q^2)$  is defined by

$$R_5(q^2) = (R_+(q^2) - R_-(q^2))/2 \quad (4)$$

and

$$R_{\pm}(q^2) = c^2 \int d^2x \exp(iqx) \exp(\pm 4\pi \Delta_m(x)), \quad (5)$$

In [8] we showed that for  $q^2 \neq 0$ ,

$$R_+(q^2) = 1 + \frac{1}{2\pi^2} (g^2/q^2)^2 + (g^2/q^2)^3 \frac{1}{\pi^3} (12 + 4 \ln[\pi g^2/q^2]) + o((g^2/q^2)^4), \quad (6)$$

and

$$R_-(q^2) = (g^2/q^2)^4 \frac{1}{\pi^4} (6.96 + 3.79 \ln[\pi g^2/q^2]) + o((g^2/q^2)^5). \quad (7)$$

Thus the cross section in DIS has no  $o(g^2/q^2)$  correction to the leading order result. We will use this and the parton cross sections presented next to evaluate the NLO parton distribution functions.

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<sup>4</sup>The prefactor  $c$  is a normal ordering dependent constant and it equals  $c = \frac{g\gamma}{(2\pi)^{3/2}}$  when the normal ordering mass equals  $m_h$ .

The parton model calculation of DIS is done perturbatively in  $g$  where the final states that are summed over consist of quanta of  $\psi$  and of  $A^\mu$ . The vector field  $A^\mu$  has asymptotic propagating states because we have chosen to regulate the IR collinear divergences (the same fermion mass singularities that occur in perturbative QCD<sub>4</sub> calculations) by temporarily giving the  $A^\mu$  field a mass,  $m_g$ . At leading order in  $g$  the only process that contributes is

$$f(k_1) + \psi(p_1) \rightarrow f(k_2) + \psi(p_2),$$

and the cross section is given in [7] to be <sup>5</sup>

$$d\hat{\sigma}^0 = \frac{e^4}{4q^4} \frac{dk_2}{E_1} \delta(1-z). \quad (8)$$

where  $z \equiv -q^2/(2p_1 \cdot q)$ .

As in QCD<sub>4</sub>, the NLO  $g^2$  corrections come from the interference of the one loop corrections with the leading order process,  $d\hat{\sigma}_v^1$ , and from gluon bremsstrahlung,  $d\hat{\sigma}_b^1$ . We choose to regularize by giving the “gluon” a mass  $m_g$ . The calculational steps that go into evaluating  $d\hat{\sigma}_v^1$  are almost the same as in those outlined in [8]. The result, written in terms of  $\beta \equiv m_g^2/q^2$ , is found to be

$$d\hat{\sigma}_v^1 = \frac{e^4}{4q^4} \frac{dk_2}{E_p} \delta(1-z) \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln[-\beta]. \quad (9)$$

The calculation of the bremsstrahlung<sup>6</sup> cross section is a little more involved and warrants a brief description. The process is

$$f(k_1) + \psi(p_1) \rightarrow f(k_2) + \psi(p_2) + A^\mu(p_3),$$

and the corresponding cross section is directly calculated to be

$$d\hat{\sigma}_b^1 = \frac{e^4}{4q^4} \frac{dk_2}{E_p} \frac{g^2}{q^2} \frac{1}{2\pi} \frac{(-8q^4)}{(p_1 - p_3)^2 (p_2 - p_3)^2} p_1 \cdot p_2 \frac{dp_2}{E_2} \frac{dp_3}{E_3} \delta^2(k_1 + p_1 - k_2 - p_2 - p_3). \quad (10)$$

It is an exercise in kinematics to do the integrals over  $p_2$ ,  $p_3$  and rewrite this cross section in terms of the two independent variables  $q^2$  and  $z$ . The result is found to be

$$d\hat{\sigma}_b^1 = \frac{e^4}{4q^4} \frac{dk_2}{E_p} \frac{g^2}{q^2} \frac{1}{\pi\beta} \frac{1}{1-z}. \quad (11)$$

For this radiation correction it is important to note that the condition

$$(p_1 + q)^2 \geq m_g^2, \quad (12)$$

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<sup>5</sup>Throughout this paper, we denote the cross-sections of the partonic subprocess, the parton model calculation, and the exact calculation by  $\hat{\sigma}$ ,  $\bar{\sigma}$ , and  $\sigma$  respectively.

<sup>6</sup>2-dimension gauge bosons have no physical degree of freedom, however, after regularization, the Schwinger model photon obtains a longitudinal mode. This makes the model much more like QCD. This method has proven successful in the calculation of lepton-antilepton annihilation process. ([8])

implies

$$\frac{1}{1-\beta} \geq z. \quad (13)$$

Now we are in a position to fold the partonic cross sections into a hadronic result. First notice that the mass of the hadron is of order  $g$ , so we cannot neglect this mass in the next-to-leading order calculation. We will follow the well-established method to deal with the hadron mass[10]. We define, without loss of generality, the hadron momentum fraction  $y$  carried by the parton as

$$p_{1+} = yP_+, \quad (14)$$

where

$$p_1^2 = 0, \quad q_+ = q_0 + q_1, \quad q_- = q_0 - q_1.$$

It follows that  $z = \xi/y$ , where  $\xi$  is the Nachtmann scaling variable

$$\xi \equiv \frac{2x}{1 + \sqrt{1 - 4x^2 m_h^2/q^2}} \quad (15)$$

Also the familiar Bjorken scaling variable  $x$  is defined to be

$$x \equiv -\frac{q^2}{2P \cdot q}, \quad (16)$$

The hadronic cross section in the parton model is then given by multiplying the underlying partonic cross sections by the parton distribution functions, both given as a functions of  $\xi$ ,  $y$ , and  $q^2$ , and integrating over  $y$  over its allowed limits. Thus

$$d\bar{\sigma} = \sum_i \int_0^1 dy f_i(y, q^2) d\hat{\sigma}(y, q^2). \quad (17)$$

The partonic cross sections in equations (8),(9), and (11) must be summed to give  $d\hat{\sigma}$  to the desired order. Further charge conjugation invariance of the underlying theory implies that the fermion and antifermion distributions are equal. Hence the sum over  $i$  in (17) is replaced by a factor of 2. Finally, the leading order result  $f = 1$  is all that is needed in evaluating the integral over  $y$  with the NLO cross sections  $d\hat{\sigma}_v^1$  and  $d\hat{\sigma}_b^1$ . The only subtlety in the integration over  $y$  occurs in the integration limits for the bremsstrahlung cross section. There equation (13) implies that

$$(1-\beta)\xi \leq y \leq 1. \quad (18)$$

Setting the partonic result equal to the exact result and keeping terms up to order  $g^2/q^2$  gives

$$f(y, q^2) = 1 - \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln\left[\frac{1-y}{y}\right]. \quad (19)$$

This is the full NLO distribution function for Schwinger model partons in the DIS process. Recall that in QCD<sub>4</sub> the NLO distribution functions may be characterized in terms of the Altarelli-Parisi equations [11]. These differential equations describe the running of the NLO distribution functions with  $q^2$  due to the collinear divergence of the radiative gluon corrections. The initial conditions for the distribution functions must come from a comparison with experimental data at some fixed  $q_0^2$ . In the calculations of this section, we have effectively solved for both the running and the initial conditions by directly relating the cross sections.

Observe that the momentum sum rule is *not* satisfied by these NLO distributions. Indeed,

$$2 \int_0^1 dy y f(y, q^2) = 1 + \frac{g^2}{m_g^2} \times (-\infty). \quad (20)$$

From this we conclude that, in the massive QED<sub>2</sub>-gluon regularization scheme, the gluon has a distribution whose expansion in  $g$  starts at order  $g^2/q^2$ . To calculate this distribution from DIS would require going to order  $g^4/q^4$  since the contributing cross section is itself of order  $g^2/q^2$ . Instead, we will now use the NLO distribution functions inferred from DIS to evaluate the Drell-Yan cross section.

### 3 Drell-Yan Process at Next-to-leading Order

We next use the parton model to calculate the cross-section for the Drell-Yan process,

$$h(P) + h(P') \rightarrow f(k) + \bar{f}(k') + X.$$

In the parton model it is assumed that the distribution functions  $f(x, q^2)$  used in the calculation of DIS are the same as the ones  $f^{DY}(x, q^2)$  to be used in the calculation of Drell-Yan process [12]. But, a priori, these distribution functions  $f^{DY}(x, q^2)$  with timelike momentum transfer,  $q^2 > 0$ , and  $f(x, q^2)$  for the DIS process with spacelike momentum transfer,  $q^2 < 0$ , could be different. In [7] it is shown that they are both equal to 1. However, higher order radiation might spoil this equality. In fact, the exchange of soft gluons between the initial state hadrons is often cited as a possible complication in even applying the parton model to the Drell-Yan process. Nevertheless, we will verify in this section, that the NLO parton model cross section, with the initial state radiation included, equals the exact cross section at NLO.

The cross section to leading order in  $e^2$  but exact in  $g^2$  is given in [7] to be

$$d\sigma = \frac{e^4}{q^4} \frac{1}{[(P \cdot P')^2 - g^4/\pi^2]^{\frac{1}{2}}} dk dk' R((q - P - P')^2), \quad (21)$$

where

$$\begin{aligned} R(q^2) &= R_+(q^2) + R_-(q^2) \\ &= c^2 \int d^2x \exp(ikx) \cosh(4\pi\Delta_m(x)). \end{aligned} \quad (22)$$

Then using the results of the asymptotic expansion in equations (6,7) we see that the exact Drell-Yan cross section is

$$d\sigma = \frac{e^4}{q^4} \frac{dkdk'}{P \cdot P'} (1 + \mathcal{O}(\frac{g^4}{q^4})). \quad (23)$$

To evaluate the NLO parton model, Drell-Yan cross section we must evaluate the underlying partonic cross sections and then fold them together with the parton distributions in the incident hadrons. At leading order the parton process is

$$\psi(p) + \bar{\psi}(p') \rightarrow f(k) + \bar{f}(k'),$$

and the cross section is evaluated in [7] to be

$$d\hat{\sigma}_{DY}^0 = \frac{e^4}{2q^4} dkdk' \delta^2(p + p' - k - k'). \quad (24)$$

To go to NLO we need to evaluate the radiative corrections to the leading order cross section. As with DIS, there are two corrections. One from the interference of the one-loop amplitudes with the leading order process,  $d\hat{\sigma}_{DY,v}^1$ . And the other from the emission of a final state gluon from one of the incoming partons,  $d\hat{\sigma}_{DY,b}^1$ .

The calculation of the virtual cross section follows the same lines as the calculations of the virtual corrections to DIS - given in the preceding section - and of the virtual corrections to  $f - \bar{f}$  annihilation described in [8]. The result is that

$$d\hat{\sigma}_{DY,v}^1 = d\hat{\sigma}_{DY}^0 \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln |\beta|. \quad (25)$$

The other order  $g^2/q^2$  correction comes from the process

$$\psi(p) + \bar{\psi}(p') \rightarrow f(k) + \bar{f}(k') + A^\mu(r),$$

and the corresponding cross section is immediately calculated to be

$$d\hat{\sigma}_{DY,b}^1 = \frac{e^4}{2q^4} dkdk' \delta^2(k + k' + r - p - p') \frac{dr}{E_r} \frac{g^2}{q^2} \frac{1}{\pi} \frac{q^4}{(p' - r)^2 (p - r)^2}. \quad (26)$$

These partonic cross sections are then folded together with the NLO distribution functions to get the NLO parton model cross-section as follows

$$\begin{aligned} d\bar{\sigma} &= \sum_{i,j} \int_0^1 dy dy' f_i(y) f_j(y') d\hat{\sigma}_{i,j}(y, y') dkdk' \\ &= 2 \int_0^1 dy dy' f(y) f(y') [d\hat{\sigma}_{DY}^0 + d\hat{\sigma}_{DY,v}^1 + \int dr \frac{d\hat{\sigma}_{DY,b}^1}{dr}]. \end{aligned} \quad (27)$$

In the second line of equality, we have used charge conjugation invariance to replace the sum over parton species with a factor of 2. Also, the outgoing gluon momentum must be

integrated over in the bremsstrahlung contribution. We will evaluate the contributions from the three cross sections separately.

For the first two terms:

$$\begin{aligned}
& 2 \int_0^1 dy dy' f(y) f(y') \left( d\hat{\sigma}_{DY}^0 + d\hat{\sigma}_{DY,v}^1 \right) \\
&= \frac{e^4}{q^4} dk dk' \int_0^1 dy dy' \delta^2(q - p - p') \left\{ 1 - \frac{g^2}{q^2} \frac{1}{\pi\beta} \left[ \ln \frac{1-y}{y} + \ln \frac{1-y'}{y'} \right] + \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln \beta \right\} \quad (28)
\end{aligned}$$

where

$$p_+ = yP_+, \quad p'_- = y'P'_-.$$

The integration over  $y$  and  $y'$  can be easily done using the following identity:

$$\delta^2(q) = 2\delta(q_+)\delta(q_-) \quad (29)$$

and we get:

$$\begin{aligned}
& 2 \int_0^1 dy dy' f(y) f(y') \left( d\hat{\sigma}_{DY}^0 + d\hat{\sigma}_{DY,v}^0 \right) \\
&= \frac{e^4}{q^4} \frac{1}{PP'} dk dk' \left[ 1 - \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln \left( \frac{2PP' - 2Pq - 2P'q + q^2}{q^2\beta} \right) \right] \quad (30)
\end{aligned}$$

Finally, consider the contribution due to the bremsstrahlung cross section,  $d\hat{\sigma}_{DY,b}^1$ . We need only the leading order distribution function, and at this order, we can neglect the effect of the mass of the hadron, but now the integral is complicated by the extra integral over the gluon momentum,  $r^\mu$ . Using equation (29), we can do the integrals over  $y$  and  $y'$  first, and after some algebras, get:

$$2 \int_0^1 dy dy' f(y) f(y') \int dr \frac{\hat{\sigma}_{DY,b}^1}{dr} = \frac{e^4}{q^4} \frac{1}{PP'} dk dk' \frac{g^2}{q^2} \frac{1}{\pi\beta} \int \frac{dr}{E_r} \quad (31)$$

We are left with the integral over the momentum  $r$ . The limits of the integration are determined by the conditions that  $y < 1$  and  $y' < 1$  which imply that

$$-\frac{(P'_- - q_-)^2 - m_g^2}{2(P'_- - q_-)} \leq r \leq \frac{(P_+ - q_+)^2 - m_g^2}{2(P_+ - q_+)} \quad (32)$$

Therefore, the contribution to the parton model Drell-Yan cross section due to gluon emission is given by

$$2 \int_0^1 dy dy' f(y) f(y') \int dr \frac{\hat{\sigma}_{DY,b}^1}{dr} = \frac{e^4}{q^4} \frac{1}{PP'} dk dk' \frac{g^2}{q^2} \frac{1}{\pi\beta} \ln \left( \frac{2PP' - 2Pq - 2P'q + q^2}{q^2\beta} \right) \quad (33)$$

Comparing equations (30) and (33), we find that the infinity in the parton distribution function cancels with that in Drell-Yan parton cross section, and the net NLO correction to the Drell-Yan cross-section in the parton model calculation vanishes:

$$d\hat{\sigma}_{DY} = d\sigma_{DY} = \frac{e^4}{q^4} \frac{1}{PP'} dk dk' \quad (34)$$

That is, the NLO parton model calculation of the Drell-Yan cross section, using the NLO distribution functions inferred from the DIS process, equals the exact Drell-Yan cross section evaluated to next-to-leading order.



## 4 Discussion and Conclusion

Simple dimension counting in  $\text{QCD}_4$  distinguishes the perturbative corrections according to their twist. Typically, one focuses on the perturbative large logarithmic corrections and neglects the higher twist corrections that are suppressed by powers of  $1/q^2$ . In contrast, the solvable  $1+1$ -dimensional massless Schwinger model coupling constant  $g$  has the dimensions of mass so that all of the corrections are suppressed by powers of  $g^2/q^2$ . Thus the perturbative corrections can not be cleanly separated from corrections due to hadron mass, interference and hadronization, as well as potential higher twist terms. We have exploited this difference to study these effects in the parton model.

It turns out the  $\xi$  scaling handles the hadron mass correctly in this model. However, it is kind of a surprise that we get a Drell-Yan cross section which agrees precisely with the exact solution without introducing multi-parton densities. There is still much to understand in this model.

The calculations in this paper establish the universality of the distribution functions in the Schwinger model at next-to-leading order. Using the asymptotic expansion of the exact results we can test whether this process-independence persists to even higher orders. Other assumptions of the model can also be tested. For example, in [8] we argued that the  $(g^2/q^2)^4 \ln[g^2/q^2]$  term in the expansion of the exact annihilation cross section was an effect of hadronization that can not be calculated from perturbation theory (see equation (7)). In  $\text{QCD}_4$  it is tacitly assumed that these hadronization effects are suppressed by inverse powers of the large squared-momentum transfer. Another fundamental assumption of the parton model is that the interference of different partonic processes is suppressed by inverse powers of large squared-momentum transfer. For example, this assumption is built into the starting point of those proofs of various factorization theorems that rely on Landau-Cutovsky cut diagrams [13]. At least in one case, we can test this assumption in the Schwinger model.

First, separate the density in equation (2) that couples to the scalar photon into two parts:

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L. \quad (35)$$

Let us use  $a$  and  $b$  to represent the annihilation operators for the quarks and antiquarks, respectively. Then

$$\begin{aligned} \bar{\psi}_L\psi_R &\sim a_L^\dagger a_R + b_R^\dagger b_L + \dots \\ \bar{\psi}_R\psi_L &\sim a_R^\dagger a_L + b_L^\dagger b_R + \dots \end{aligned} \quad (36)$$

Next, without loss of generality, assume the hadron is moving right. Then the contribution of the first term of equation (35) to DIS process corresponds to a quark coming out of the hadron and the second term corresponds to an antiquark coming out the hadron. In parton model, the cross sections for these two process are summed over. In the exact solution, however, we can calculate the interference of these two terms using

$$\begin{aligned} \bar{\psi}_L\psi_R &= \frac{c}{2} : \exp(2i\sqrt{\pi}h) \\ \bar{\psi}_R\psi_L &= \frac{c}{2} : \exp(-2i\sqrt{\pi}h) \end{aligned} \quad (37)$$

Thus, it turns out the inference term is proportional to  $R_-(q^2)$  and from equation (7) the leading order of this interference occurs at  $o(g^8/q^8)$ . This term is indeed is very small but is nonvanishing. A similar analysis applies to Drell-Yan process.

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